# ROTATIONAL MECHANICS\_PART-III\_(Toppling, Collisions and Miscellaneous Situations) PART-A LEVEL-I\_(THEORY)

hysics Sir 🌙

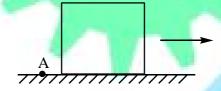
Simplifying learning Physics

#### ONE OR MORE THAN ONE ANSWER CORRECT

1. A particle of mass m at a distance r from a fixed point is attracted towards the fixed point by a force given by equation  $F = k/r^2$ , where k is a positive constant. If L be its angular momentum with respect to the fixed point, which of the following equations is correct?

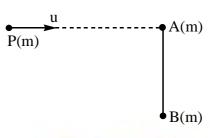
A) 
$$\Delta \left(\frac{L^2}{mr^2} - \frac{k}{r}\right) = 0$$
  
B)  $\Delta \left(\frac{1}{2}m\left(\frac{dr}{dt}\right)^2 + \frac{k}{r} + \frac{L^2}{2mr^2}\right) = 0$   
C)  $\Delta \left(\frac{1}{2}m\left(\frac{dr}{dt}\right)^2 - \frac{k}{r}\right) = 0$   
D)  $\Delta \left(\frac{1}{2}m\left(\frac{dr}{dt}\right)^2 - \frac{k}{r} + \frac{L^2}{2mr^2}\right) = 0$ 

2. A block of mass 'm' moves on a horizontal rough surface with initial velocity 'v'. The height of the centre of mass of the block is 'h' from the surface. Consider a point 'A' on horizontal surface which lies in vertical plane passing through c.m of cube.



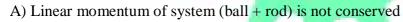
- A) Angular momentum about 'A' is mvh initially
- B) About point 'A' angular momentum of the block decreases during sliding
- C) Torque of the forces acting on block is zero about A
- D) Angular momentum is not conserved about A
- 3. A particle collides with a uniform smooth rod at rest lying on smooth horizontal plane. Initially the velocity of particle is not along the length of rod. Then the ratio of the velocity of centre of rod to it's angular velocity will
  - A) depend on the coefficient of restitution
  - B) depend on masses of particle and that of rod
  - C) depend on initial velocity of particle
  - D) depend on the length of rod and the position of point where the particle strikes

4. Two particles A and B, of mass m each, are joined by a rigid massless rod of length *l*. A particle P of mass m, moving with a speed u normal to AB, strikes A and sticks to it. The centre of mass of the 'A + B + P' system is C.



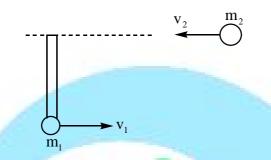
- A) The velocity of C before impact is u/3
- B) The velocity of C after impact is u/3
- C) The velocity of 'A + P' immediately after impact is u/2
- D) The velocity of b immediately after impact is zero
- 5. In the given figure a ball strikes a rod at its end elastically and rod is hinged smoothly at point A. Then which of the statement(s) is/are correct (regarding just before and after the collision)

////<sup>A</sup>//////



- B) Angular momentum of system (ball + rod) about hinged point A is conserved
- C) Kinetic energy of ball is conserved
- D) Linear momentum of ball is conserved

6. Two girls Sita and Gita are skating towards each other on smooth ice along parallel lines as shown in figure. The distance between the lines is *l*. The mass of two girls are  $m_1$  and  $m_2(m_1 < m_2)$  and their respective velocities are  $v_1$  and  $v_2(v_2 > v_1)$ . One of the girls holds a stick of length *l* and negligible mass. When the girls pass each other, second girl grasps the stick and the girls move together, each of them on either side of the stick. Mark the correct statements -



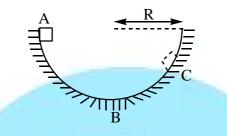
A) In the centre of mass reference frame of system both the girls have momentum of equal magnitudeB) Due to torque of normal reaction between hands of girl grasping rod, angular momentum of system about centre of mass increases

C) The girls start moving towards each other by pulling the stick, angular velocity of system will increase

D) After the girl has grabbed the rod system rotates in anticlockwise sense while centre of mass translates towards left

- 7. Rod inclined at an angle  $\theta$  with vertical strikes the horizontal frictionless ground.  $(\theta \neq 0)$ . The angular velocity of rod before striking is zero & linear velocity v is vertically downwards. Which of the following statements is/are correct?
  - A) Angular momentum of rod is conserved during collision about point of impact on ground
  - B) For elastic collision the centre of rod rebounds back with velocity v
  - C) For elastic collision the centre of mass of rod rebounds back with velocity v
  - D) For perfectly inelastic collision the velocity of centre of mass just after collision becomes zero

8. A small cube with edge length "a" is placed in a fixed rough concave surface of radius R(a << R) as shown. Initially, placing the block at point A, is released. After sometime the block crosses the lowest point B and reaches the highest point C on the other side. Considering that the block does not topples anywhere and also only one swing from A to C, which of the following is/are correct ?



A) As long as the block is on left the total angular displacement of the block about its centre is clockwise

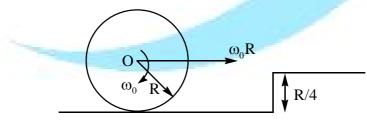
B) As long as the block is on right the total angular displacement of the block about its centre is anticlockwise

C) In entire motion from A to C the total angular displacement of the block about centre is anticlockwise

D) In entire motion from A to C the total angular displacement of the block about centre of concave surface is anticlockwise

#### PASSAGE

A solid cylinder of mass m and radius R is rolling without slipping on a horizontal surface with angular velocity  $\omega_0$ . The velocity of centre of mass of cylinder is  $\omega_0 R$ . The cylinder comes across a step of height R/4. (Assume required friction is present at edge of step.) Answer the following questions based on above information.



- 9. Find the minimum angular velocity  $(\omega_0)$  of cylinder so that cylinder rides up the step without slipping or jumping.
  - A)  $\frac{2}{5}\sqrt{\frac{3g}{R}}$  B)  $\frac{3}{5}\sqrt{\frac{3g}{R}}$  C)  $\frac{5}{2}\sqrt{\frac{3g}{R}}$  D)  $\sqrt{\frac{g}{3R}}$

10. Find the maximum angular velocity  $(\omega_0)$  of cylinder so that cylinder rides up the step without slipping or jumping.

A)  $\frac{3}{5}\sqrt{\frac{3g}{R}}$  B)  $\frac{2}{5}\sqrt{\frac{3g}{R}}$  C)  $\frac{1}{2}\sqrt{\frac{3g}{R}}$  D)  $\frac{5}{3}\sqrt{\frac{3g}{R}}$ 

#### PASSAGE

It is Diwali time and we would like to crack a few crackers for you. But since we cannot do that in this paper, we would ask you a few questions on Diwali crackers. A popular cracker is Sudarshan chakra or Zameenchakkar. It is in form of a uniform disc free to rotate about a fixed vertical axis passing through it's centre. As we ignite at free end P, the explosive on periphery burns and expels gases with a constant speed relative to the disc along the tangent. The rate at which mass burns is constant. The speed of ejected gases relative to the disc is u. The ejected gases exert a thrust force on the disc which rotates the disc. Assume that radius of cracker decreases very slowly and at any time it remains in shape of uniform disc. Assume initial radius of the chakra is R.

- 11. Choose the **incorrect** statement(s)
  - A) the torque on the disc goes on increasing
  - B) the torque on the disc remains constant
  - C) the thrust force goes on increasing
  - D) the thrust force remains the same

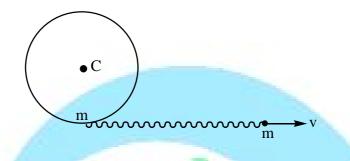
12. Choose the **correct** option(s)

A) The angular acceleration of the disc varies with radius x of disc as (C is constant)  $\frac{C}{2}$ 

- B) The angular acceleration of the disc varies with radius x of disc as (C is constant)  $\frac{C}{3}$
- C) The angular velocity of disc when radius reduces to R/2 is  $\frac{4u}{R}$
- D) The angular velocity of disc when radius reduces to R/2 is  $\frac{u}{2R}$

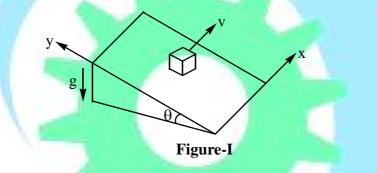
#### **INTEGER TYPE**

- 13. A particle of mass is m is attached to a disc of equal mass m by means of a slack ideal string as shown. The disc is hinged about its centre C, on a horizontal smooth table. The particle is projected with initial velocity v along the tangent of sphere. If its speed just after the string becomes taut is
  - $\frac{Kv}{3}$ , find the value of K.



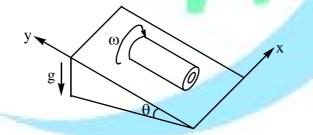
#### MATCHING

14. Match the direction of forces of column – A with their non-zero component in column – B.



At the moment shown when block on incline has velocity perpendicular to line of greatest slope parallel to x-axis.

Cylinder rolls without slipping on incline such that its axis remains parallel to line of greatest slope.

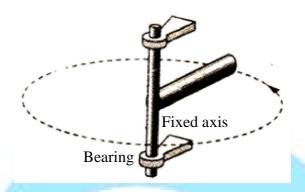


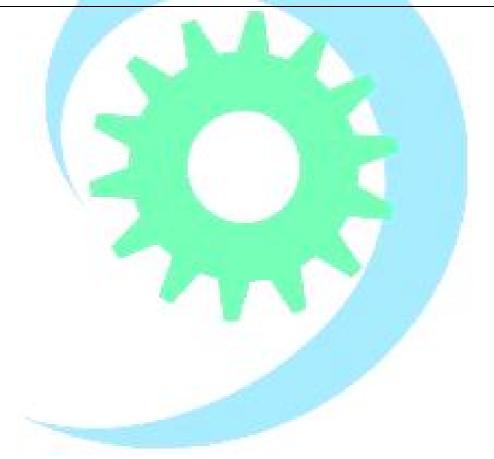
**Figure-II** 

	Column-A_(Direction of force)		Column-B_(Non-zero component)			
(a)	Friction for figure-I at the instant shown	(p)	Only z			
(b)	Friction after small time for figure-I	(q)	Only y			
(c)	Friction for figure-II	(r)	Both x and y			
(d)	Torque due to normal contact force about	(s)	Only x			
	centre of mass for figure-II					

#### SUBJECTIVE TYPE

15. A cylinder rotates with angular speed  $\omega$  about an axis through one end, as in figure. Choose an appropriate origin and show qualitatively the vectors  $\vec{L}$  and  $\vec{\omega}$ . Are those vectors parallel? Do symmetry considerations enter here?



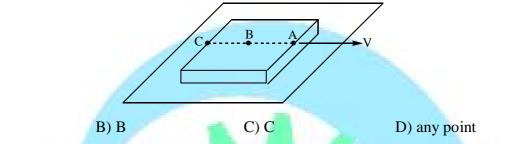


## <u>PART-B</u> <u>LEVEL-II\_(APPLICATION)</u>

#### SINGLE ANSWER CORRECT

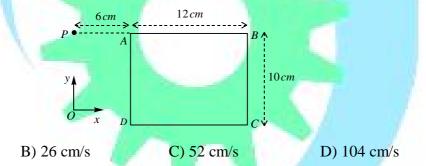
A) A

A rectangular box of uniform density moving on a rough horizontal surface towards right as shown.
 A, B and C are three points on the surface, lies on central line of box as shown. The tendency of toppling of box is least, when a point mass m (less than the mass of the box) is fixed at

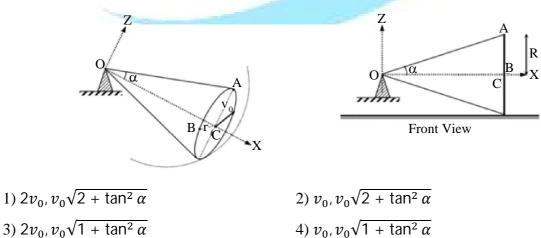


2. At an instant, the point P in the figure represents the instantaneous centre of rotation for a uniform rectangular plate ABCD ( $12 \ cm \times 10 \ cm$ ) at the given instant the point A (corner) has velocity 24 cm/

sec  $(-\hat{j})$ . The magnitude of the velocity of the centre of mass of the plate is

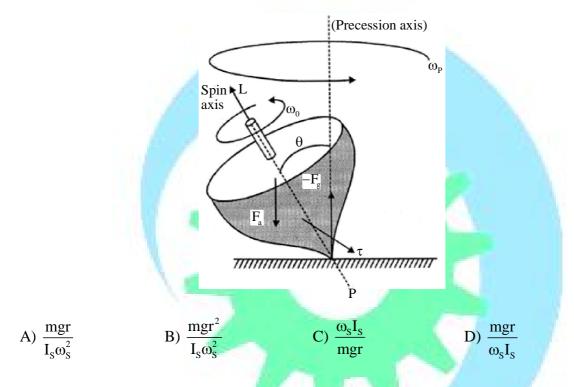


- A) 13 cm/s
- 3. A cone rolls without slipping on the horizontal surface so that its apex remains fixed and the axis of cone is horizontal. The radius of base of cone is r and the semi-vertex angle is α. The centre of base of cone is C and it moves with constant speed v<sub>0</sub>. Find the speed of point A and point B. (Angle ACB is right angle). Point A and B lie on circumference of the base of cone.

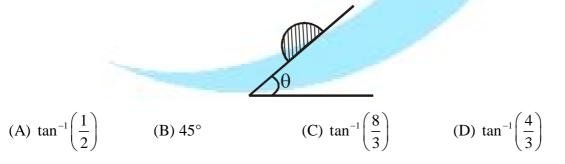


4. Precession is the result of the angular velocity of rotation (spin) and the angular velocity produced by the torque. It is an angular velocity about a line that makes an angle  $\theta$  with the permanent rotation axis. In the diagram, a top of mass m is performing precession 7motion. The torque due to gravitational force  $F_g$  causes a change in the angular momentum L in the direction of that torque causing the top to process with angular velocity  $\omega_p$  which is given by :

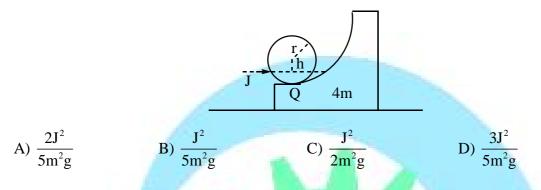
(Given r is the distance of COM from point of rotation 'P'.  $I_s \& \omega_s$  are the moment of inertia and angular velocity of top about spin axis respectively.)



5. An uniform hemi-solid sphere is placed with flat surface on rough inclined plane as shown in figure. If friction is large for no sliding, then the minimum angle  $\theta$  at which toppling occur is

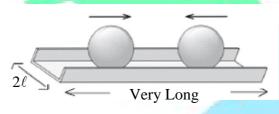


6. A wedge of mass 4m is placed at rest on a smooth horizontal surface. A uniform solid sphere of mass m and radius r is placed at rest on the flat portion of the wedge at a point Q as shown in the figure. A sharp horizontal impulse J is given to the sphere at a point below h = 0.4 r from the centre of the sphere. The radius of curvature of curved portion on the wedge is R. The coefficient of friction to the left side of point Q is  $\mu$  and to the right side of point Q is zero. The maximum height to which the centre of mass of the sphere will climb on the curved portion of the wedge is



#### ONE OR MORE THAN ONE ANSWER CORRECT

7. Two identical billiard balls (uniform solid spheres) each of radius R and mass m moving with a speed  $V_0$  roll without slipping towards each other on a horizontal U-shaped trough that is sufficiently deep such that the balls are clear of its base (see figure). The width of the trough is  $2\ell$ . The resulting collision is perfectly elastic and during it each ball reverses its linear velocity though their angular velocities are not effected. The surface of trough is not smooth



Mark the **CORRECT** statement (s)

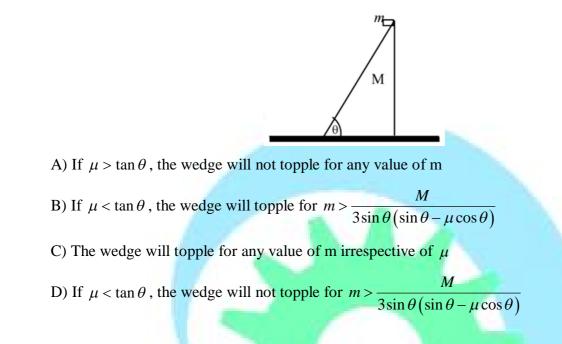
A) Initial magnitude of angular velocity of each ball is  $\frac{V_0}{P}$ .

B) Including the first collision, the balls will collide more than once if  $\ell > \sqrt{\frac{3}{5}} R$ 

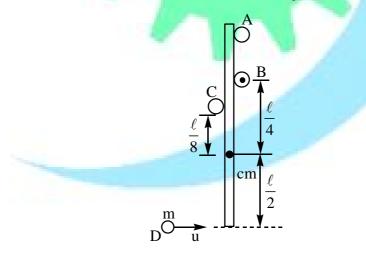
C) Between the end of first collision to the start of second collision, the angular momentum of the system of each ball is constant about any point on the center line of the horizontal plane containing the top edges of the trough.

D) If second collision occurs for a given value of  $\ell$ , the value of linear speed of each ball just before second collision is independent of coefficient of friction between trough and sphere surfaces

8. Friction between the wedge shown and the horizontal floor is sufficient to prevent the wedge from sliding. The mass of the wedge is M and its angle of inclination is  $\theta$ . A small block of mass m is just placed near the top of the wedge and released. Friction coefficient between the wedge and the block is  $\mu$ 

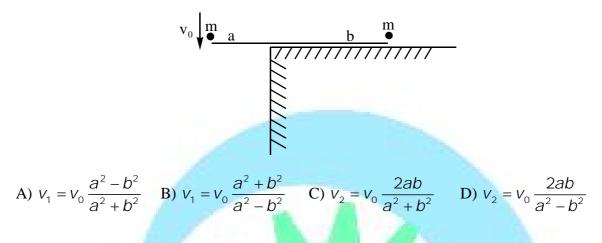


9. Mass of all the balls and uniform rod of length *l* is m and the system of balls A, B, C and rod was initially at rest on horizontal frictionless ground. A, B, C are touching rod but not sticking to it. Ball D collides with one end of rod perpendicularly. Which of the following is correct statement about direction of motion of small balls A, B, C, D & rod shown just after collision of ball D with rod?

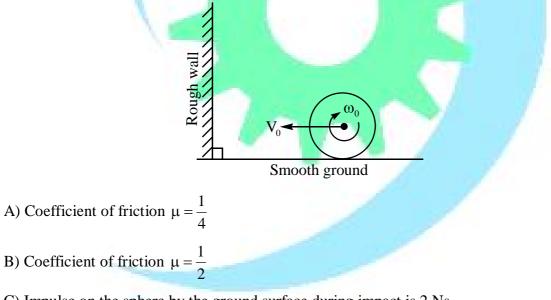


- A) A, B moves towards right just after collision
- B) C moves left just after collision
- C) D moves towards right just after collision
- D) Centre of mass of rod moves towards right just after collision

10. A massless stick lies on a table, with a length a hanging over the edge and a length b on the table, as shown in figure. A ball with mass m (ball 2) lies on the stick at its right end. Another ball with m (ball 1) is dropped above the left end hits the end with speed  $V_0$ . Assuming that all interactions in the setup are elastic, what are the velocities of the two balls right after the collision?

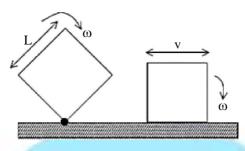


11. A solid sphere of mass 2 kg and radius 1m has linear velocity  $V_0 = 4 \text{ m/s}$  and angular velocity  $\omega_0 = 9 \text{ rad/s}$  as shown. It collides elastically with a rough wall with coefficient of friction  $\mu$ . If the sphere after colliding with the wall immediately rolls without slipping in opposite direction then



- C) Impulse on the sphere by the ground surface during impact is 2 Ns
- D) Impulse on the sphere by the ground surface during impact is 4 Ns

12. There is a cube of mass M and side L. It is hinged at one of its edge as shown in figure. If initially it is kept as shown in figure, and a very slight clockwise impulse is given to it so that it first starts toppling (w = 0) about hinge, it begins to rotate clockwise. When the face of the cube strikes the ground, it immediately comes to rest. Then



A) The angular velocity of the cube just before it strikes the ground is  $\sqrt{\frac{3}{2}} \frac{g}{L} (\sqrt{2} - 1)$ 

B) The total impulse in the vertical direction that has acted on the cube during the collision with the

ground is  $M\sqrt{\frac{3gL(\sqrt{2}-1)}{8}}$ 

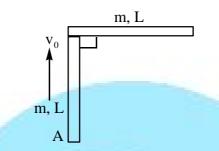
C) The impulse of torque about the hinged edge that has acted on the cube during the collision is

$$\sqrt{\frac{2M^2L^3g\left(\sqrt{2}-1\right)}{3}}$$

D) The impulse of torque about the hinged edge that has acted on the cube during the collision is

$$\sqrt{\frac{3}{2}}M^{2}L^{3}g\left(\sqrt{2}-1\right)$$

13. A thin uniform horizontal rod of mass m, length L moves on a smooth horizontal surface with a horizontal velocity  $v_0$  in the direction of its length. One end of the rod hits at one end of a second horizontal uniform rod of same length and same mass, kept at rest on the same surface, perpendicular to the first rod. At the moment they collide, the two rods stick together and the angle between them remains  $90^{\circ}$ . Select the correct option(s) from below.



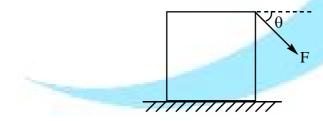
A) Angular velocity of the system of two rods after collision about an axis perpendicular to the plane of rods and passing through the centre of mass of combination, is  $3v_0 / 5L$ 

B) Maximum velocity attained by the end A of rod after collision is  $\frac{V_0}{2} \left[ 1 + \frac{3}{\sqrt{10}} \right]$ 

C) Time taken to attain maximum velocity first time after collision by the end A of rod is  $\frac{5L}{3V_0}\tan^{-1}(3)$ 

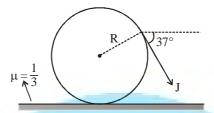
D) Velocity of centre of mass of system is  $V_0/2$ 

14. A cube of side 'a' and mass m lies on a horizontal surface. Cube is pulled by force F acting at an angle  $\theta$  as shown in figure. If  $F_{min}$  is the minimum force and  $\theta$  is the corresponding angle made by force with horizontal so that cube topples without sliding,

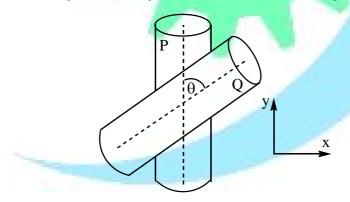


A) For 
$$\mu = \frac{1}{3}$$
, value of  $\theta = 45^{\circ}$  and  $F_{min} = mg\sqrt{2}$   
B) For  $\mu = \frac{1}{3}$ , value of  $\theta = 45^{\circ}$  and  $F_{min} = mg/\sqrt{2}$   
C) For  $\mu = \frac{2}{3}$ , value of  $\theta = 0^{\circ}$  and corresponding  $F_{min} = mg/2$   
D) For  $\mu = \frac{3}{4}$ , value of  $\theta = 0^{\circ}$  and  $F_{min} = mg/2$ 

15. A unknown solid sphere of mass m and radius r is kept on rough horizontal surface with coefficient of friction  $\mu = \frac{1}{3}$  as shown in the figure. An impulse  $J = MV_0$  is applied tangentially on the sphere at an angle 37° with the horizontal as shown in the figure. (Assuming J >>Mgdt, g = 10 m/s<sup>2</sup>)

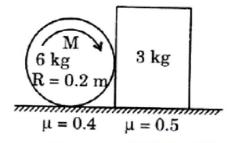


- (A) The velocity of centre of mass of the sphere just after impulse applied is  $V_0$
- (B) The angular velocity of sphere just after impulse applied is  $\frac{2V_0}{P}$
- (C) The velocity of centre of mass of the sphere just after impulse applied is  $\frac{4V_0}{5}$
- (D) The angular velocity of sphere just after impulse applied is  $\frac{V_0}{P}$
- 16. A cylinder P of radius  $r_p$  is being rotated at a constant angular velocity  $\omega_p \hat{j}$  with the help of a motor about its axis that is fixed. Another cylinder Q of radius  $r_q$  free to rotate about its axis that is also fixed is touched with and pressed on P making an angle  $\theta$  between their axes. Soon after the cylinders are pressed against each other, a steady state is reached and the cylinder Q acquires a constant angular velocity. What can you conclude when the steady state is reached?



- A) Angular velocity of cylinder Q is  $\vec{\omega}_Q = -\frac{\omega_P r_P}{r_Q \cos \theta} \left( \sin \theta \hat{i} + \cos \theta \hat{j} \right)$
- B) Angular velocity of cylinder Q is  $\vec{\omega}_Q = -\frac{\omega_P r_P \cos \theta}{r_Q} \left(\sin \theta \hat{i} + \cos \theta \hat{j}\right)$
- C) Unit vector of frictional force on cylinder Q is  $(\sin\theta \hat{i} + \cos\theta \hat{j})$
- D) Frictional force on each cylinder becomes vanishingly small

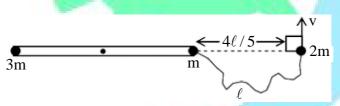
17. A clockwise torque of 6 N-m is applied to the circular cylinder as shown in the figure. There is no friction between the cylinder and the block.



- A) The cylinder will be slipping but the system does not move forward
- B) The system cannot move forward for any torque applied to the cylinder
- C) The acceleration of the system will be  $1 \text{ m/s}^2$  forward
- D) The angular acceleration of the cylinder is  $10 \text{ rad} / \text{s}^2$

#### PASSAGE

A light and thin rod of length  $\ell$  is lying on horizontal table. Two small balls of masses 3m and m are fixed on two ends of a massless rod. The rod is free to rotate about fixed vertical axle passing through midpoint of rod. A third ball of mass 2m is fastened to ball of mass m through a string of length  $\ell$  as shown in figure. The ball is projected horizontally with speed v along Y axis as shown. When the string becomes taut, impulsive forces will act on the system at that instant. We can refer to this event as a collision.



- 18. Which of the following is true at the moment just after the string becomes taut.
  - A) The angular velocity of rod is 18v/59L

B) The angular velocity of rod is 13v/57L

- C) The component of velocity of the third ball of mass 2m along Y axis is 41v/59
- D) The component of velocity of the third ball of mass 2m along Y axis is 41v/57
- 19. If impulse of tension in string during collision is J, and impulse of the force exerted by axle on the rod during collision is N, then which of the following is correct

A) J = 60mv/59 B) J = 58 mv/57 C) J/N = 10/9 D) J/N = 30/31

#### PASSAGE

20.

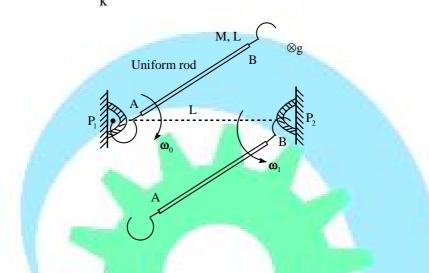
21.

22.

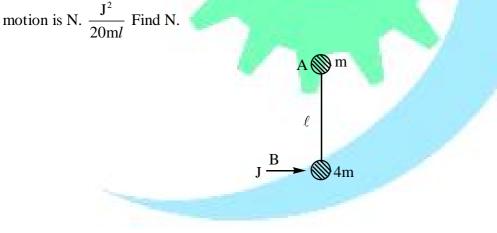
A homogeneous bar of mass M rotates uniformly in alternating directions to and fro around the horizontal axis going through its center, while a ball of mass m bounces between the ends of the bar in vertical plane as shown in the figure. The angular velocity of the bar is  $\pm \frac{2\pi}{3}$  rad sec<sup>-1</sup> and the time period of the periodic motion of the ball is 2 second. (The mechanical loss and the duration of the collisions are negligible.) What is the velocity of the ball at the highest point of its orbit? D)  $5\sqrt{2}$  m/s B)  $5\sqrt{3}$  m/s A) 10 m/s C) 5 m/s How long is the bar? D)  $5\sqrt{2}$  m B)  $10\sqrt{3}$  m A)  $5\sqrt{3}$  m C) 10 m What is the  $\frac{m}{M}$  ratio ? B)  $\frac{\sqrt{3}}{10}$ D)  $\frac{\pi\sqrt{3}}{9}$ π√3 A)  $\frac{\pi}{18}$ 

#### **INTEGER TYPE**

23. A uniform rod AB of mass M, length L is rotating about hinge P<sub>1</sub> with angular velocity  $\boldsymbol{\omega}_0$  on a smooth horizontal surface as shown in the diagram, at the end A and B there are two light hooks having negligible size. End B of the rod collides with hinge P<sub>2</sub> and rod starts rotating with angular velocity  $\boldsymbol{\omega}_1$  about hinge P<sub>2</sub> while hinge P<sub>1</sub> and end A leave contact smoothly immediately after collision. If  $\boldsymbol{\omega}_1$  is of the form  $\frac{\boldsymbol{\omega}_0}{k}$ . Then the value of k will be?

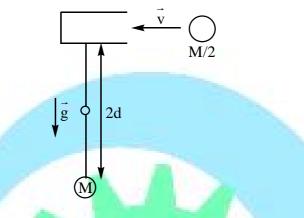


24. An inextensible string connects two masses m and 4m and the system rests on a smooth horizontal floor. An impulse J is imparted to B as shown in figure. The tension in the string in the subsequent

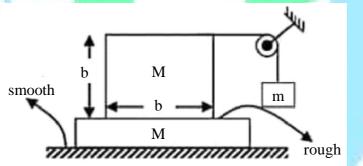


25. A pendulum consists of a ball of mass 1 kg attached to the end of a rigid bar of length 3 m which is pivoted at the center. At the other end of the bar is a container ("catch"). A second ball of mass  $\frac{1}{2}$  kg

is thrown into the catch at a velocity 6 m/s where it sticks. For this problems, ignore the mass of the pendulum bar and catch, and treat the balls as if they were point masses (i.e. neglect their individual moments of inertia). Find the total mechanical energy (in J) lost when the incoming ball struck in the catch.

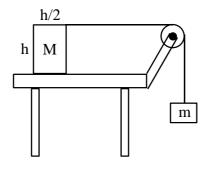


- 26. A block of mass M placed on another block of mass M, is connected to a hanging body of mass m by an inextensible string which passes over a smooth fixed pulley. If the ground is smooth, find the ratio
  - of  $\frac{m}{M}$  so as to just topple the block. (assume no slipping between both the blocks)



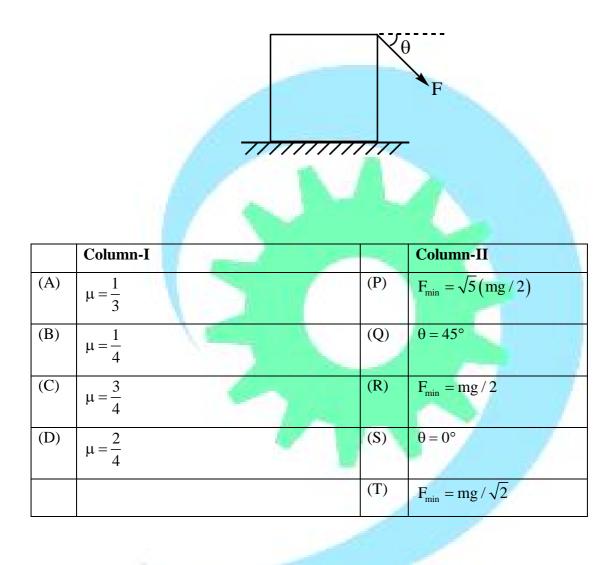
27. A cylinder of height h, diameter  $\frac{h}{2}$  and mass M and with a homogeneous mass distribution is placed on a horizontal table. One end of a string running over a pulley is fastened to the top of the cylinder, a body of mass m is hung from the other end and the system is released.

Friction is negligible everywhere. At what minimum ratio  $\frac{m}{M}$  will the cylinder tilt?



#### MATCHING

28. A cube of side a and mass m lies on a horizontal surface. Cube is pulled by force F acting at an angle  $\theta$  as shown in figure. If  $F_{\min}$  is the minimum force and  $\theta$  is the corresponding angle made by force with horizontal so that cube topples without sliding, then match the value of  $F_{\min}$  and  $\theta$  in column II with values of coefficient of friction  $\mu$  between cube and surface in column I



1	2	3	4	5	6	7	8	9	10
D	ABD	D	ABCD	AB	ACD	А	BCD	А	А
11	12	13	14	15					
ABC	BC	2	a - s, b - r, c - q, d - s	Not always, yes					

### PART-A\_LEVEL-I\_(THEORY)

## PART-B\_LEVEL-II\_(APPLICATIONS)

1	2	3	4	5	6	7	8	9	10
С	С	A	D	С	А	BCD	AB	CD	AC
11	12	13	14	15	16	17	18	19	20
AD	ABC	ABCD	BCD	AB	BC	CD	AC	AC	В
21	22	23	24	25	26	27	28		
В	D	2	4	6	1	57	A – QT, B – P, C – RS, D – RS		